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Energy of a Charged Wormhole

Mustafa Saltı[1] and Oktay Aydogdu[1,2]

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The Møller energy(due to matter and fields including gravity) distribution of the traversable Lorentzian wormhole space-time by the scalar field or electric charged is studied in two different approaches of gravity such as general relativity and tele-parallel gravity. The results are found exactly the same in these different approximations. The energy found in tele-parallel gravity is also independent of the tele-parallel dimensionless coupling constant, which means that it is valid in any tele-parallel model. Our results sustains that (a) the importance of the energy-momentum definitions in the evaluation of the energy distribution of a given space-time and (b) the viewpoint of Lassner that the Møller energy-momentum complex is a powerful concept of energy and momentum.

KEY WORDS: Energy; charged wormhole.

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1. INTRODUCTION

In literature, after Einstein's expression (Einstein, 1915) for the energy and momentum distributions of the gravitational field, many attempts have been proposed to resolve the gravitational energy problem (Bergmann and Thomson, 1953; Landau and Lifshitz, 2002; Møller, 1958; Mikhail *et al.*, 1993; Papapetrou, 1948; Qadir and Sharif, 1992; Tolman, 1934; Vargas, 2004; Weinberg, 1972). Except for the definition of Møller, these definitions only give meaningful results if the calculations are performed in "Cartesian" coordinates. Møller constructed an expression which enables one to evaluate energy and momentum in any coordinate system. In general relativity, Virbhadra (1999) using the energy and momentum complexes of Einstein, Landau-Lifshitz, Papapetrou and Weinberg for a general non-static spherically symmetric metric of the Kerr-Schild class, showed that all of these energy-momentum formulations give the same energy distribution

[1] Department of Physics, Faculty of Art and Science, Middle East Technical University, 06531, Ankara, Turkey; e-mail: musts6@yahoo.com
[2] To whom correspondence should be addressed at Department of Physics, Faculty of Art and Science, Middle East Technical University, 06531, Ankara, Turkey; e-mail: oktay231@yahoo.com

as in the Penrose energy-momentum formulation. Further, several examples of particular space-times have been investigated and different energy-momentum pseudo-tensor are known to give the same energy distribution for a given space-time (Chamorro and Virbhadra, 1996; Radinschi, 2000a,b; Ragab, 2005; Vagenas, 2003; Virbhadra, 1990a,b, 1992, 1997a,b; Virbhadra and Parikh, 1993; Virbhadra *et al.*, 1996; Virbhadra *et al.*, 1998).

The problem of energy-momentum localization has also been considered in alternative gravitation theory, namely tele-parallel gravity (Mikhail *et al.*, 1993; Vargas, 2004). In Gen. Relat. Gravit. 36, 1255 (2004); Vargas, using the definitions of Einstein and Landau-Lifshitz in tele-parallel gravity, found that the total energy is zero in Friedmann-Robertson-Walker space-times and his result is the same as those calculated in general relativity (Banerjee and Sen, 1997; Johri *et al.*, 1995; Rosen, 1994). After this interesting work, Salti *et al.* (2005a,b,c; Aydogdu and Salti, 2005; Salti and Havare, 2005) considered the viscous Kasner-type and the Bianchi-type I space-times, and the results obtained in tele-parallel gravity are the same as obtained in general relativity. In another work, Aydogdu (to appear) shows that the energy distribution of the universe, based on the locally rotationally symmetric Bianchi-type II space-time, is same in both tele-parallel gravity and general relativity.

In this paper, we obtain the energy distribution(due to matter plus fields) associated with a charged wormhole solution using the general relativity and the tele-parallel gravity version of Møller's energy-momentum definition.

Notations and conventions: $c = h = 1$, metric signature $(+, -, -, -)$, Greek indices and Latin ones run from 0 to 3. Throughout this paper, Latin indices $(i, j, \ldots)$ represent the vector number, and Greek indices $(\mu, \nu, \ldots)$ represent the vector components.

2. THE TRAVERSABLE WORMHOLE SOLUTIONS

A search for the traversable wormhole solutions with realistic matter has been for long, and is still remaining to be, one of the most intriguing challenges in gravitational studies. One of attractive features of wormholes is their ability to support electric or magnetic "charge without charge" (Wheeler, 1957) by letting the lines of force thread from one spatial asymptotic to another.

As it is widely known, traversable wormhole can only exist with exotic matter sources, more precisely, if the energy-momentum tensor of the matter source of gravity violates the local and averaged null energy condition (Hochberg and Visser, 1997)

$$T_{\mu\nu}k^{\mu}k^{\nu} \geq 0, \qquad k_{\mu}k^{\nu} = 0 \tag{1}$$

Scalar fields are able to provide good examples matter needed for wormholes: on the one hand, in many particular models they do exhibit exotic properties, on the other, many exact solutions are known for gravity with scalar sources. We will consider some examples of wormhole solutions.

2.1. The Static Wormhole

The static wormhole space-time without and charge is given by Kim and Lee (2001)

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -e^{2\Xi(r)}dt^2 + \frac{dr^2}{1-\frac{\xi(r)}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{2}$$

Here the arbitrary functions $\Xi(r)$, $\xi(r)$ are defined as the lapse and wormhole shape functions, respectively. The shape of the wormhole is determined by $\xi(r)$. There can be two requirements about the wormhole function $\xi(r)$ for the wormhole in order to be maintained. These are positiveness and flatness conditions. If $r \to \infty$, $\xi(r)$ approaches $2M$ that is defined as the mass of wormhole (Visser, 1995). Therefore $\xi(r)$ should be defined as the positive function, and the condition $r > \xi(r)$ which means the existence of the minimum radius also support the positiveness of $\xi(r)$. The other condition comes from the flare-out condition of the shape of the wormhole. When the proper distance $\zeta\epsilon(-\infty, +\infty)$ is defined as $d\zeta = dr/(1-\xi(r))$, the condition should be

$$\frac{d^2\zeta}{dr^2} > 0 \qquad \Rightarrow \qquad \frac{\xi - r\xi'}{\xi^2} > 0. \tag{3}$$

2.2. Wormhole with Scalar Field

If we put $\xi_{ef} = \xi - \rho/r$ instead of ξ with $\Xi = 0$, thus the effect by the scalar field on the wormhole is simply by represented as the change of the wormhole function ξ into $\xi - \rho/r$ without any interaction term. Here ρ depends on the changing rate of the scalar field φ and play the role of scalar charge. From this point of view, the metric of the wormhole space-time with the scalar field should be (Kim and Lee, 2001)

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^2 + \frac{dr^2}{1+\frac{\xi(r)}{r}+\frac{\rho}{r^2}} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{4}$$

Since the components are always positive, there is no horizon in this space-time. For the dimensional reasons, the wormhole function has the form given below.

$$\xi = \xi_0^{2\omega+1} r^{1/2\omega+1} \tag{5}$$

where ω is the proper parameter of equation of state (Kim, 1996), and this should be less than $-\frac{1}{2}$ so that the exponent of r can be negative to satisfy the flareness condition. The shape of the effective wormhole will vary with the value of this parameter and ρ via the additional factor $-\rho/r$.

2.3. Wormhole with Electric Charge

The procedures of the case of electric charge is the same as the case of scalar field. The space-time model for a wormhole with electric charge is given by Kim and Lee (2001)

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -\left(1 - \frac{Q}{r^2}\right)dt^2 + \frac{dr^2}{1 - \frac{\xi(r)}{r} + \frac{Q}{r^2}} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (6)$$

This line-element is the combination of Morris-Thorne type spherically symmetric static wormhole and Reissner-Nordström space-time. If $Q = 0$, the space-time model is reduced the Morris-Thorne wormhole and when $\xi = 0$, it transforms the Reissner-Nordström black hole with zero mass.

3. MØLLER'S ENERGY-MOMENTUM DEFINITION

3.1. Energy in General Relativity

In general relativity, the energy-momentum complex of Møller (Bergmann and Thomson, 1953; Landau and Lifshitz, 2002; Møller, 1958; Papapetrou, 1948; Qadir and Sharif, 1992; Tolman, 1934; Weinberg, 1972) is given by

$$M_{\mu}^{\nu} = \frac{1}{8\pi}\Sigma_{\mu,\alpha}^{\nu\alpha} \quad (7)$$

satisfying the local conservation laws:

$$\frac{\partial M_{\mu}^{\nu}}{\partial x^{\nu}} = 0 \quad (8)$$

where the antisymmetric super-potential $\Sigma_{\mu}^{\nu\alpha}$ is

$$\Sigma_{\mu}^{\nu\alpha} = \sqrt{-g}[g_{\mu\beta,\gamma} - g_{\mu\gamma,\beta}]g^{\nu\gamma}g^{\alpha\beta}. \quad (9)$$

The locally conserved energy-momentum complex M_{μ}^{ν} contains contributions from the matter, non-gravitational and gravitational fields. M_0^0 is the energy density and M_a^0 are the momentum density components. The momentum four-vector of Møller is given by

$$P_{\mu} = \int\int\int M_{\mu}^{0}\,dx\,dy\,dz. \quad (10)$$

Using Gauss's theorem, this definition transforms into

$$P_{\mu} = \frac{1}{8\pi}\int\int \Sigma_{\mu}^{0\alpha}\mu_{\alpha}dS. \quad (11)$$

where μ_α (where $\alpha = 1, 2, 3$) is the outward unit normal vector over the infinitesimal surface element dS. P_i give momentum components P_1, P_2, P_3 and P_0 gives the energy.

3.2. Energy in the Tele-parallel Gravity

The tele-parallel theory of gravity(tetrad theory of gravitation) is an alternative approach to gravitation and corresponds to a gauge theory for the translation group based on Weitzenböck geometry (Weitzenböck, 1923). In the theory of tele-parallel gravity is attributed to torsion (Hayashi and Shirafuji, 1978), which plays the role of a force (de Andrade and Pereira, 1997), and the curvature tensor vanishes identically. The essential field is acted by a nontrivial tetrad field, which gives rise to the metric as a by-product. The translational gauge potentials appear as the nontrivial item of the tetrad field, so induces on space-time a tele-parallel structure which is directly related to the presence of the gravitational field. The interesting place of tele-parallel theory is that, due to its gauge structure, it can reveal a more appropriate approach to consider some specific problem. This is the situation, for example, in the energy and momentum problem, which becomes more transparent.

Møller modified general relativity by constructing a new field theory in tele-parallel space. The aim of this theory was to overcome the problem of the energy-momentum complex that appears in Riemannian Space (Møller, 1961, 1978). The field equations in this new theory were derived from a Lagrangian which is not invariant under local tetrad rotation. Saez (1983) generalized Møller theory into a scalar tetrad theory of gravitation. Meyer (1982) showed that Møller theory is a special case of Poincare gauge theory (Hayashi and Shirafuji, 1980a,b; Hehl *et al.*, 1980).

In tele-parallel gravity, the super-potential of Møller is given by Mikhail *et al.* (1993; Vargas, 2004) as

$$U_\mu^{\nu\beta} = \frac{(-g)^{1/2}}{2\kappa} P_{\chi\rho\sigma}^{\tau\nu\beta} \left[\Phi^\rho g^{\sigma\chi} g_{\mu\tau} - \lambda g_{\tau\mu} \gamma^{\chi\rho\sigma} - (1 - 2\lambda) g_{\tau\mu} \gamma^{\sigma\rho\chi}\right] \tag{12}$$

where $\gamma_{\alpha\beta_\mu} = h_{i\alpha} h^i_{\beta;\mu}$ is the con-torsion tensor and $h_i{}^\mu$ is the tetrad field and defined uniquely by $g^{\alpha\beta} = h_i^\alpha h_j^\beta \eta^{ij}$ (here η^{ij} is the Minkowski space-time). κ is the Einstein constant and λ is free-dimensionless coupling parameter of tele-parallel gravity. For the tele-parallel equivalent of general relativity, there is a specific choice of this constant.

Φ_ρ is the basic vector field given by

$$\Phi_\mu = \gamma^\rho{}_{\mu\rho} \tag{13}$$

and $P_{\chi\rho\sigma}^{\tau\nu\beta}$ can be found by

$$P_{\chi\rho\sigma}^{\tau\nu\beta} = \delta_{\chi}^{\tau} g_{\rho\sigma}^{\nu\beta} + \delta_{\rho}^{\tau} g_{\sigma\chi}^{\nu\beta} - \delta_{\sigma}^{\tau} g_{\chi\rho}^{\nu\beta} \tag{14}$$

with $g_{\rho\sigma}^{\nu\beta}$ being a tensor defined by

$$g_{\rho\sigma}^{\nu\beta} = \delta_{\rho}^{\nu}\delta_{\sigma}^{\beta} - \delta_{\sigma}^{\nu}\delta_{\rho}^{\beta}. \tag{15}$$

The energy-momentum density is defined by

$$\Pi_{\alpha}^{\beta} = U_{\alpha,\lambda}^{\beta\lambda} \tag{16}$$

where comma denotes ordinary differentiation. The energy is expressed by the surface integral;

$$E = \lim_{r\to\infty} \int_{r=constant} U_0^{0\zeta} \eta_\zeta \, dS \tag{17}$$

where η_ζ (with $\zeta = 1, 2, 3$) is the unit three-vector normal to surface element dS.

4. CALCULATIONS

In this part of the study, we will calculate the energy distribution(due to matter and fields including gravity) for a general metric that includes three wormhole models given above. The general line-element is

$$g_{\mu\nu} dx^\mu dx^\nu = -B^2 dt^2 + A^2 dR^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{18}$$

4.1. In Genaral Relativity

The matrix of the $g_{\nu\mu}$ is defined by

$$\begin{pmatrix} B^2 & 0 & 0 & 0 \\ 0 & -A^2 & 0 & 0 \\ 0 & 0 & -R^2 & 0 \\ 0 & 0 & 0 & -R^2\sin^2\theta \end{pmatrix} \tag{19}$$

and its inverse $g^{\mu\nu}$ is

$$\begin{pmatrix} B^{-2} & 0 & 0 & 0 \\ 0 & -A^{-2} & 0 & 0 \\ 0 & 0 & -\frac{1}{R^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{R^2\sin^2\theta} \end{pmatrix}. \tag{20}$$

For the line-element (18) under consideration we calculate

$$\Sigma_0^{01} = \frac{B_r r^2 \sin\theta}{\sqrt{AB}} \tag{21}$$

which is the only required component of $\Sigma_\mu^{\nu\alpha}$ for our purpose. Here, the index r indicates derivative with respect to r. Using this expression in equation (11) we obtain the energy distribution

$$E = \frac{B_r r^2 \sin\theta}{2\sqrt{AB}} \tag{22}$$

4.2. In Tele-parallel Gravity

The general form of the tetrad, h_i^μ, having spherical symmetry was given by Robertson (1932). In the Cartesian form it can be written as

$$\begin{aligned} &h_0^0 = iA, \quad h_a^0 = Cx^a, \quad h_0^\alpha = iDx^\alpha, \\ &h_a^\alpha = B\delta_a^\alpha + Ex_\alpha x^\alpha + \epsilon_{a\alpha\beta}Fx^\beta \end{aligned} \tag{23}$$

where $A, B, C, D, E,$ and F are functions of t and $r = \sqrt{x^\alpha x^\alpha}$, and the zeroth vector h_0^μ has the factor $i^2 = -1$ to preserve Lorentz signature[3].

Using the general coordinate transformation

$$h_{a\mu} = \frac{\partial \mathbf{X}^\nu}{\partial \mathbf{X}^\mu} h_{a\nu} \tag{24}$$

where $\mathbf{X}^\mu$ and $\mathbf{X}^\nu$ are, respectively, the isotropic and Schwarzschild coordinates (t, r, θ, ϕ), we obtain the tetrad $h_a{}^\mu$ as

$$\begin{pmatrix} \frac{i}{B} & 0 & 0 & 0 \\ 0 & \frac{1}{A}\sin\theta\cos\phi & \frac{1}{r}\cos\theta\cos\phi & -\frac{\sin\phi}{r\sin\theta} \\ 0 & \frac{1}{A}\sin\theta\sin\phi & \frac{1}{r}\cos\theta\sin\phi & \frac{\cos\phi}{r\sin\theta} \\ 0 & \frac{1}{A}\cos\theta & -\frac{1}{r}\sin\theta & 0 \end{pmatrix} \tag{25}$$

where $i^2 = -1$. Hence, the required non-vanishing component of $U_\mu^{\nu\beta}$ is (TCI Software Research, 1998; Wolfram Research, 2003)

$$U_0^{01} = \frac{B_r r^2 \sin\theta}{\kappa\sqrt{AB}} \tag{26}$$

[3] The tetrad of Minkowski space-time is $h_a^\mu = \mathrm{diag}(i, \delta_a^i)$ where $(i = 1,2,3)$.

Substituting this result in energy integral, we have the following energy distribution

$$E = \frac{B_r r^2}{2\sqrt{AB}}. \tag{27}$$

This is the same energy distribution as obtained in general relativity.

4.3. Special Cases for the Energy

In the previous two parts of this section, we obtained the general energy distribution in a wormhole solution with extra field such as scalar and electric fields. Now, we find the energy for some special cases given in Section 2.

- The static wormhole, to get the limit of the general metric, we can write $B = e^{2\Xi(r)}$ and $A = r/(r - \xi(r))$ in the line-element (18). Therefore the energy distribution (due to matter and fields including gravity) in this case is obtained as

$$E = r^{\frac{3}{2}}(r - \xi)^{\frac{1}{2}} e^{\Xi}\Xi_r \tag{28}$$

For this line element, taking the limits of $\Xi(r) \to 0$ and $\Xi(r) \to 0$, thus the model transforms into the Minkowski space-time, and the energy goes to zero.

- Wormhole with scalar field, to reduce our general metric into this case, we should write $B = 1$ and $A = r^2/(r^2 - r\xi(r) + \rho(r))$ in the line-element (18). Using this expressions, the energy of this case is found as

$$E = 0 \tag{29}$$

If we take the limits of $\xi(r) \to 0$ and $\rho(r) \to 0$, the model gives the Minkowski space-time, and the energy is also zero.

- Wormhole with electric charge, taking $B = (1 + \frac{Q^2}{r^2})$ and $A = (1 - \frac{\xi}{r} + \frac{Q^2}{r^2})^{-1}$ into the line-element (18) we obtain the wormhole solution with electric charge. Considering these definitions, the energy is obtained as given below.

$$E = \frac{-Q}{r}\left(\frac{r^2 - r\xi + Q^2}{1 + \frac{r^2}{Q^2}}\right)^{\frac{1}{2}} \tag{30}$$

This model also includes the Minkowski space-time, under the limits of $Q \to 0$ and $\xi(r) \to 0$, and the energy distribution of the model vanishes.

5. DISCUSSIONS

We evaluated the energy distributions for some wormhole solutions with extra fields such as scalar field and electric charge using both the general relativity and the tele-parallel gravity versions of Møller's energy-momentum definition. The energy distributions were found the same in both of these different approximations of gravity. The energy distribution obtained in tele-parallel gravity is also independent of the tele-parallel dimensionless coupling constant, which means that it is valid in any tele-parallel model. We also taken some limits of the wormhole models considered to show they includes the Minkowski metric, and find the zero energy as we expect.

Furthermore, this paper sustains (a) the importance of the energy-momentum definitions in the evaluation of the energy distribution of a given space-time, (b) the viewpoint of Lassner that the Møller energy-momentum complex is a powerful concept of energy and momentum, (c) the opinion that different energy-momentum expressions definitions could give the same result in a given space-time and (d) the Møller energy-momentum definition allows to make calculations in any coordinate system.

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